Thermalization of gluons and onset of collectivity at RHIC due to $gg \leftrightarrow ggg$ interactions

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A number of promising results of a new 3+1 dimensional Monte Carlo parton cascade including inelastic multiplication processes ($gg \leftrightarrow ggg$) are elaborated: (1) thermalization and chemical saturation; (2) the onset of longitudinal hydrodynamical expansion; (3) the build up of elliptic flow. We first briefly outline the basic idea of the algorithm. Full simulations are done with initial conditions for the kinetic partons via minijets or with ones stemming from a color glass condensate. The inclusion of the inelastic channels leads to a very fast kinetic equilibration and also to an early creation of pressure.

I. INTRODUCTION AND BRIEF DESCRIPTION OF THE CASCADE

The prime intention for present ultrarelativistic heavy ion collisions at CERN and at Brookhaven lies in the possible experimental identification of a new state of matter, the quark gluon plasma (QGP). Measurements [1] at RHIC of the elliptic flow parameter v_2 for nearly central collisions suggest that - in comparison to fits based on simple ideal hydrodynamical models - the evolving system builds up a sufficiently early pressure and potentially also achieves (local) equilibrium.

On the other hand, the system in the reaction is at least initially far from any equilibrium configuration. To microscopically describe and understand the dynamics of ultrarelativistic heavy ion collisions, and to address the crucial question of thermalization and early pressure buildup, we have developed a kinetic parton cascade algorithm [2] inspired by perturbative QCD including for the first time inelastic ('Bremsstrahlung') collisions $gg \leftrightarrow ggg$ besides the binary elastic collisions.

It is well known, that a parton cascade analysis, incorporating only elastic (and forward directed) $2 \leftrightarrow 2$ collisions described via one-gluon exchange, shows that thermalization and early (quasi-)hydrodynamical behaviour (for achieving sufficient elliptic flow) can not be built up or maintained, but only if a much higher, constant and isotropic cross section $\sigma_{eff} \approx 45$ mb is being employed [3]. By employing such a high cross section, however, especially for the very early phase when the system is very dense, the physical justification of a quasiclassical kinetic transport equation becomes unwarranted: The mean free path $\lambda_{mfp} \approx 1/(n\sigma_{eff}) \sim 1/n$, whereas the mean distance among the gluons is $d = 1/(n)^{1/3}$. n denotes the number density of gluons. If σ_{eff} stays constant at such a large value and if the gluon density is getting large, as typically achieved in the early stages of the reaction, one has the unphysical picture that the mean free path would be much smaller than the interparticle distance $\lambda_{mfp} \ll d$. The use of a semiclassical, kinetic Boltzmann transport description is, from a classical point of view, unjustified and not valid. In quantum mechanical terms, the gluons as dynamical degrees of freedom would aquire a collional width $\Gamma \approx n\sigma_{eff}v_{rel} \approx 1/\lambda_{mfp}$ being then much larger than the typical energy $E \approx 3T \sim 1/d$, and thus would resemble very broad excitations and are not quasi-particles by any means. On the other hand, typical Debye screened pQCD cross section scale roughly like the inverse temperature squared, $\sim 1/T^2$, so that λ_{mfp} does not become smaller than d, but is of the same order and slightly larger. This is also true for inelastic channels, as long as the coupling stays small enough.

In addition, the possible importance of the inelastic reactions on overall thermalization was raised in the so called 'bottom up thermalization' picture [4]. It is intuitively clear that gluon multiplication should not only lead to chemical equilibration, but also should lead to a faster kinetic equilibration. This represents a further (but not all) important motivation for developing a consistent algorithm to handle also inelastic processes.

The conceptual new simulation lies in treating elastic and inelastic multiplication collisions in a unified manner [2]. Most importantly, the (multiparticle) back reation channel ($ggg \rightarrow gg$) is treated fully consistently by respecting detailed balance within the same algorithm. If the back channel would be neglected, one would possess no serious handle on soft gluon production! The system would cool too fast simply by gluon multiplication, and the gluon population ('entropy' production) would dramatically oversaturate. This states a serious problem in the older parton cascade schemes [5], where standard gluon splitting without adequate recombination is employed.

The numerical challenge is how to numerically incorporate the fusion [2]: The idea of the stochatic method is to divide the total space in sufficiently small local cells, in which the system is quasi homogenous. In this local cells master equations in momentum space are solved. Detailing on the the back reaction of $gg \leftrightarrow ggg$, we define a transition probability in a time interval with $0 \leq P_{32} \ll 1$ for a given triplet of gluons with specific momenta in a

small local cell:

$$P_{32} = \frac{\Delta N_{coll}^{3\to 2}}{\Delta N_1 \Delta N_2 \Delta N_3} = \frac{1}{8E_1 E_2 E_3} \frac{I_{32}}{N_{test}^2} \frac{\Delta t}{(\Delta^3 x)^2} \,, \tag{1}$$

where I_{32} is defined as the phase space integral

$$\frac{1}{2!} \int \frac{d^3 p_1^{'}}{(2\pi)^3 2E_1^{'}} \frac{d^3 p_2^{'}}{(2\pi)^3 2E_2^{'}} |\mathcal{M}_{123\to 1'2'}|^2 (2\pi)^4 \delta^{(4)}(p_1 + p_2 + p_3 - p_1^{'} - p_2^{'}) \,.$$

$\Delta^3 x$ denotes the volume of a specific cell and N_{test} represents the number of test-particles. The entering matrix element one obtains from the $2 \to 3$ element via a standard prefactor governed by a detailed balance relation. The latter is given by

$$|\mathcal{M}_{gg\to ggg}|^2 = \left(\frac{9g^4}{2} \frac{s^2}{(\mathbf{q}_\perp^2 + m_D^2)^2}\right)\left(\frac{12g^2 \mathbf{q}_\perp^2}{\mathbf{k}_\perp^2[(\mathbf{k}_\perp - \mathbf{q}_\perp)^2 + m_D^2]}\right)\theta(k_\perp \Lambda_g - \cosh y) \,, \tag{2}$$

where $g^2 = 4\pi\alpha_s$. $\mathbf{q}_\perp$ and $\mathbf{k}_\perp$ are the perpendicular component of the momentum transfer and that of the momentum of the radiated gluon in the c.m.-frame of the collision, respectively. y denotes the rapidity of the radiated gluon. We thus take $gg \to ggg$ in leading-order of pQCD and consider an effective Landau-Pomeranchuk-Migdal suppression with Λ_g denoting the gluon mean free path, which is given by the inverse of the total gluon collision rate $\Lambda_g = 1/R_g$, and also employ a standard screening mass m_D for the infrared sector of the scattering amplitude. Both m_D and Λ_g are calculated selfconsistently. The coupling is taken as scale dependent. If the system is undersaturated, accordingly, the cross sections than are noticeably higher than compared to the magnitude at thermal equilibrium.

Incorporating the algorithm in a full 3+1 dimensional Monte Carlo cascade, one achieves a covariant parton cascade which can accurately handle the immense elastic as well as inelastic scattering rates occuring inside the dense (gluonic) system. The important task is to develop a dynamical mesh of (expanding) cells in order to handle the extreme initial situation. For an exhaustive testing of the code we refer to the original paper [2]. Instead, in the following section, we give a number of, as we believe, important and to some extent also still rather preliminary results obtained via real 3+1-dim. simulations for RHIC energies.

II. SELECTED RESULTS OF THE CASCADE OPERATING AT RHIC

We first address the question of the importantance of the (still) pQCD inspired reactions on the thermalization and early pressure build up for heavy ion collisions at RHIC. The algorithm can incorporate any specified initial conditions for the freed on-shell partons. The first results we show take as a conservative point of view minijet initial conditions.

Minijet production comes from multiple binary nucleon-nucleon-scattering in a nucleus-nucleus-collision, where we have chosen a conservatively large transverse momentum cutoff of $p_t > p_0 = 2$ GeV/c [6], according to the differential jet cross section:

$$\frac{d\sigma_{jet}}{dp_T^2 dy_1 dy_2} = K \sum_{a,b} x_1 f_a(x_1, p_T^2) x_2 f_b(x_2, p_T^2) \frac{d\sigma_{ab}}{d\hat{t}} \,. \tag{3}$$

p_T denotes the transverse momentum and y_1 and y_2 are the rapidities of the produced partons. The partons are are distributed in space-time via the corresponding overlap function. In the original paper we have chosen no formation time for the minijet gluons, so that they can immediately interact. One can think of a phenomenological formation time $\tau_f \approx 1/p_t$ for the gluons to become onshell and freed from the nucleus wavefunction. For the first two figures we show (taken from [2]) no formation time is employed. The ones after are calculated by incorporating such a formation time. As it turns out, when doing calculation w/w.o. formation time the difference in extracting quantities is really small, after the very early formation of minijet gluons has terminated.

For a cutoff of $p_0 = 2$ GeV/c the number of initial minijets is about 900 with a $\frac{dN_g}{dy} \approx 200$ at midrapidity for central Au+Au collisions at $\sqrt{s} = 200$ AGeV. This is a rather low value, but we keep to this conservative estimate for our first two figures. In Figs. 1 we show the transverse momentum spectrum obtained with the full dynamics at (spatial) midrapidity ($\Delta\eta = 1$) at different times for partons of a central cylinder of radius $R \leq 1.5$ and, respectively, a similar calculation with including only elastic collisions. From $t = 0$ on first only gluonic minijets with $p_t > 2$ GeV are populated. Energy degradation to lower momenta proceeds rapidly by gluon emission within the first fm/c. Maintenance of (quasi-)kinetic and (later) chemical equilibrium is given up to 4 fm/c, where longitudinal and
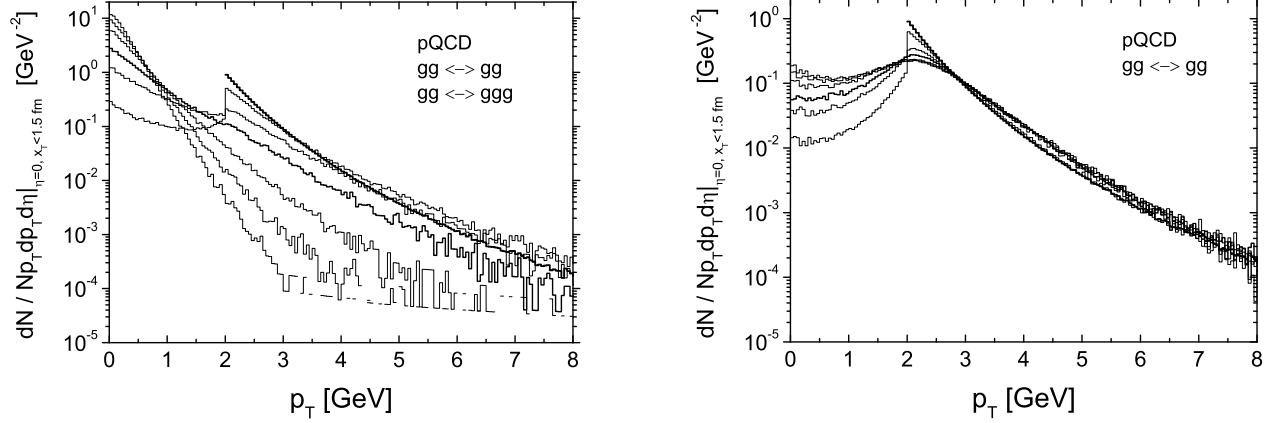
FIG. 1: Left panel: Transverse momentum spectrum at (spatial) midrapidity ($\Delta\eta = 1$) at different times (t=0.2, 0.5, 1, 2, 3 and 4 fm/c from second upper to lowest line) for a real, central fully 3-D ultrarelativistic heavy ion collision. Only the partons residing in a central cylinder of radius $R \leq 1.5$ fm are depicted. From $t = 0$ on first only gluonic minijets with $p_t > 2$ GeV are populated (most-upper, boldfolded line). Energy degradation to lower momenta proceeds rapidly by gluon emission within the first fm/c. Maintenance of (quasi-)kinetic and (later) chemical equilibrium is given up to 4 fm/c, where longitudinal and transversal (quasi-hydrodynamical) work is done resulting in a continous lowering of the temperature. Right panel: Like the left panel, but now by incorporating only elastic pQCD collisions. The most-upper and boldfolded histogram with a lower cutoff at $p_T = 2$ GeV denotes the spectrum of the primary gluons (minijets).
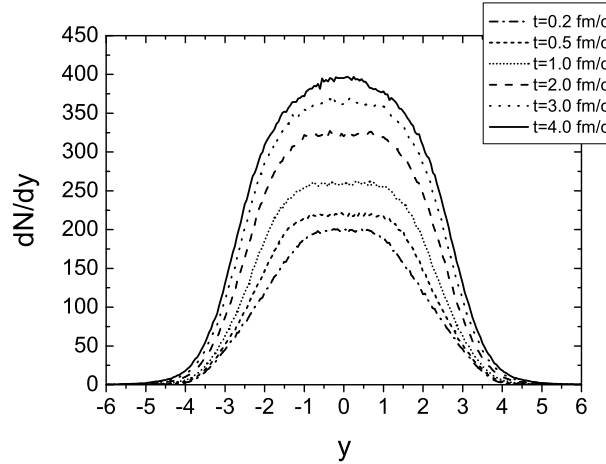

FIG. 2: Gluon number distribution versus momentum rapidity at the time $t = 0.2$, 0.5, 1.0, 2.0, 3.0 and 4.0 fm/c during the expansion.

transversal (quasi-hydrodynamical) work is done resulting in a continous lowering of the temperature. This can be seen by the continous steepening of the exponential slopes of the spectrum with progressing times. It turns out that kinetic momentum equilibration occurs at times of about 1 fm/c, whereas full chemical equilibration occurs on a smaller scale of about 2-3 fm/c. As the right panel of the figure does not show any sign of strong momentum degradation, thermalization is clearly due to the incoporation of the inelastic channels. For the complete transversal region a remedy of the initial non-equilibrium high momentum tail will remain stemming from the escaping minijets of the outer region.

An intuitive interpretation of the fast thermalization is the following: Whereas the elastic scattering is forward
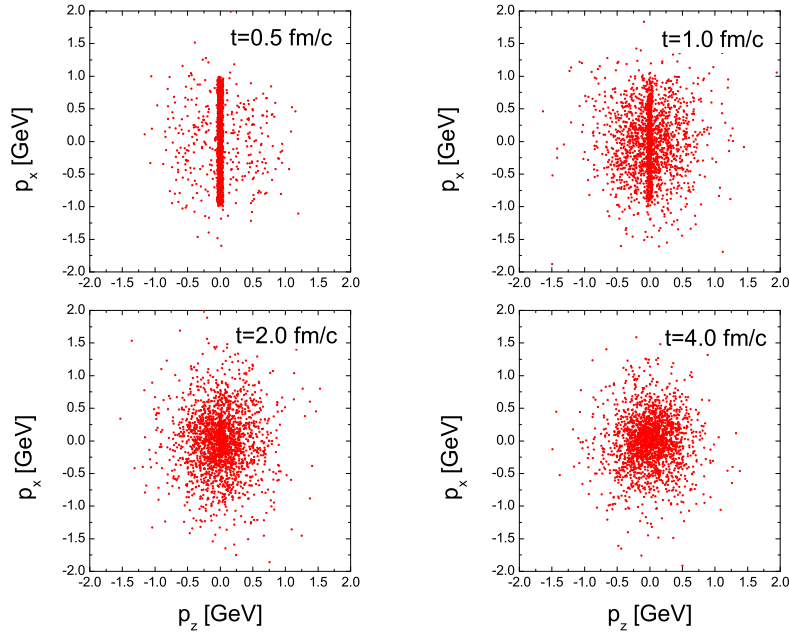
FIG. 3: 'bottom up thermalization': A typical color glass condensate initial condition is taken at $\tau_0 = 0.4$ fm/c within a Bjorken geometry. The saturation scale is taken as $Q_s = 1$ GeV. The evolution of the momentum occupation is shown for four subsequent times sampled within a space-time rapidity interval of $\Delta\eta = 0.1$ and a central transverse region of $R \leq 1.5$ fm. One clearly recognizes the population of the 'soft' gluons and a subsequent degradation of the 'hard' initial gluons.

peaked, this is not fully the case for the inelastic reaction including a LPM cutoff [2]. Especially the emitted gluons show a flat and non-forward angle distribution. This underlines why the inelastic processes are so important not only for accounting for chemical equilibration, but also for kinetic equilibration. It are these Bremsstrahlung radiations which actually bring about early thermalization to the QGP.

In Fig. 2 the ongoing production of gluons versus rapidity is given. The reason is that the system, for the momentum cutoff chosen, starts being highly undersaturated in gluon number so that abundant gluon production by Bremsstrahlung sets in right after the minijets have appeared. At the end of the evolution at t=4 fm/c about $\frac{dN_g}{dy} \approx 400$ are at midrapidity, i.e. the gluon number has doubled. If we compare the amount of transversal energy at midrapidity with the experimental factor, we are roughly below by at least a factor of 2. This means, that for the initial conditions too few gluons have been assumed.

In Fig. 3 we now give a first dynamical realization of the so called 'bottom up thermalization' scenario as advocated in [4]. The initial distribution of gluons is taken as that of a characteristic color glass condensate with the same parameters as in the simple parametrization given in [7]. The initial geometry is of Bjorken type. The evolution of the momentum occupation is shown for four subsequent times sampled within a space-time rapidity interval of $\Delta\eta = 0.1$ and a central transverse region of $R \leq 1.5$ fm. One nicely recognizes the population of the 'soft' gluons at an early time scale and a subsequent degradation of the 'hard' initial gluons. All this happens roughly within 1 fm/c. In the last picture we also see that all particles are clearly more centered around the origin, demonstrating once more the ongoing cooling and quasi hydrodynamical behaviour from 2 to 4 fm/c. As a surprise, there is one striking difference compared to the idealistic scenario of [4]: The number of gluons (per unit rapidity) is slighly decreasing, although a strong parametric enhancement has been advocated in [4] due to Bremsstrahlung production. The reason is that for the initial conditions taken from [7], a clear separation of hard and soft scale is not really given, the parametric estimate has thus taken to be with caution. In any case our exploratory study of the 'bottom up thermalization' picture is interesting in its own right and deserves further detailed analysis.

Fig. 4 summarizes our finding with respect to a potential onset of a collective, longitudinal expansion [8]. Various initial conditions are investigated. For the initial minijet scenario the cutoff scale p_0 has been lowered from 2.0 down to 1.3 GeV. (In all this calculation the minijet gluons are now formed with an initial formation time $\tau_f = 1/p_t$.)
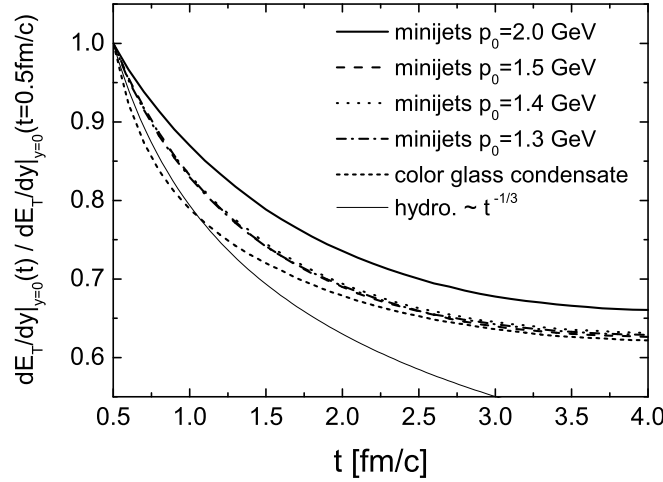
FIG. 4: Time evolution of the transverse energy per unit momentum rapidity at midrapidity for various initial minijet conditions and one color glass condensate condition of central Au+Au collisions at $\sqrt{s} = 200$ AGeV. All transverse energies are gauged according to their value at $t = 0.5$ fm/c. As a comparison the simple scaling of a longitudinally expanding ideal fluid is also given. For the first 1 to 1.5 fm/c the system clearly expands close to ideal by exhibiting longitudinal work.

The color glass condensate initial condition is taken as just described above. In the figure the time evolution of the transverse energy per unit momentum rapidity at midrapidity is depicted. All transverse energies are gauged according to their value at $t = 0.5$ fm/c. As a comparison the simple scaling of a longitudinally expanding ideal fluid is also given. For the first 1 to 1.5 fm/c the system expands close to ideal by exhibiting longitudinal work with a ongoing cooling. This nicely demonstrates that the system experiences quasi ideal hydrodynamic behaviour from $t \approx 0.5$ fm/c to $t \approx 2$ fm/c, after the gluons have started to kinetically equilibrate. If there would be no inelastic pQCD collisions, but only elastic, the transverse energy would stay almost constant ([2]). In addition, the evolution of the transverse energy seems to be only moderate sensitive to the initial condition chosen (minijets or color glass condensate), showing quite an impressive universal behaviour [8]. This behaviour is due to the presence of inelastic collisions.

One can also now seek for the 'optimal' initial conditions being in line with RHIC data: From STAR one has $dE_T/dy|_{y=0} \approx 620$ GeV for central collisions. With the conservative parameter $p_0 = 2$ GeV for the minijet distribution the final transverse energy would be smaller by a factor of 2. A similar 'underestimation' holds for the chosen color glass condensate initial condition, which thus will need some more special fine-tuning (potentially with some high momentum tail, see eg [9]) in order to fit to data. For the minijets the 'optimal' parametrisation would be choosing $p_0 \approx 1.4$ GeV. In such a situation the gluon number is always close to full saturation.

In Fig. 5 we now turn to an important benchmark, the calculation of the elliptic flow parameter v_2. These calculations are still very preliminary and no exhaustive testing has been finished. As initial conditions minijets have been taken with a cutoff $p_0 = 1.5$ GeV. We depict the evolution of $v_2(t)$ of all gluons at midrapidity ($\Delta y = 1$) for two noncentral collisions at an impact parameter b=4 fm and b=6 fm. Typically v_2 has its maximum for an impact parameter of $b = 8 - 10$ fm with $v_2 \approx 0.06$; for $b = 4$ fm one finds $v_2 \approx 0.03$ and for $b = 6$ fm $v_2 \approx 0.04$ (see eg calculations and nice compilation and comparison to STAR data in ref. [10]). The reason why we choose the smaller impact parameter is that we simply wanted to stay to the default transverse grid of subcells. For larger impact parameter a more fine tuned grid is probably needed and accordingly has to be numerically tested. We stop the evolution at times $t = 4$ fm/c as then the gluon density drops roughly below 1 fm^{-3}, i.e. hadronization should set in. One clearly sees that the flow steadily builds up to approximately 3 percent for the two impact parameters, i.e. $v_2 \approx 0.03$, but is still not saturated at a time of 4 fm/c. These values are already quite large and do come very close to the just cited 'experimental' results. For the $b = 6$ fm case the flow might be too low by 20 percent, yet $v_2(t)$ is still in the tendency of rising with progressing late times. In addition, a smaller amount of v_2 can come from the later hadronic phase [11] and/or from the very early stage of a color glass condensate [12]. It might also be conceivable that preisotropization of the particles [13] due to temporary instabilities being manifested by strong
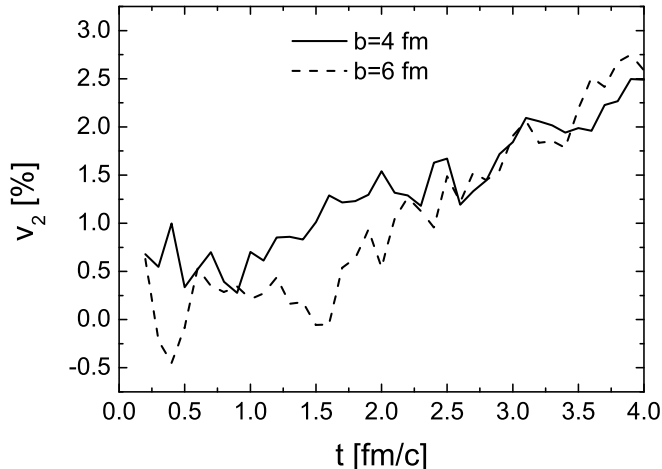
FIG. 5: First and preliminary results for extracting the elliptic flow parameter $v_2(t)$ at midrapidity ($\Delta\eta = 1$) for two noncentral Au+Au collisions at an impact parameter b=4 fm and b=6 fm. As initial conditions minijets have been taken with a cutoff $p_0 = 1.5$ GeV. One clearly sees that the flow steadily builds up to approximately 3 percent for the two impact parameters, but is still not saturated at a time of 4 fm/c.

classical chromoelectromagnetic fields also will yield some small initial elliptic flow. There is still a lot room for investigations, yet our preliminary results are very encouraging.

III.   SUMMARY AND CONCLUSIONS

The presented extensive study of a new and complex parton cascade shows that gluon multiplication via Bremsstrahlung (and absorption) is of utmost importance to understand kinetic equilibration, chemical saturation and the build up of early pressure. The existence of the latter nicely shows up in the continous steepening of the transverse momentum spectra and the build up of longitudinal work. For the various settings of initial conditions kinetic equilibration is achievd on a timescale of less than about 1 fm/c, whereas the full chemical equilibration occurs on a somewhat slower scale of about 2-3 fm/c, if the initial conditions are chosen so that the gluon number is initially undersaturated. First results have been presented with color glass condensate initial condition. The bottom up thermalization picture seems to work for a realistic coupling with a kinetic equilibration occuring again also in less than 1 fm/c. On the other hand no strong amplification in the gluon number occurs. Finally very preliminary results on the build up of elliptic flow have been shown. Maybe too early to fully claim, it seems that approximately 80 per cent of the total v_2 can be induced by the inelastic parton interactions.

Is the QGP a strongly coupled system, a sQGP, as advertised in various recent agenda, where eg high cross sections have been discussed and employed, in order to come close to ideal hydrodynamics? Our analysis is still ongoing and has to be more detailed, before claiming it can account for a variety of data. The first calculation are indeed very encouraging to proceed. The pQCD cross sections employed are typically on the order of less than 1 mb to a few mb (at the later stage of the reaction, when the system has to hadronize) [2], depending also on the degree of gluon saturation.

In the future a lot of further details have to be explored [14]: Thermalization, also of the light and heavy quark degrees of freedom, has to be investigated for various initial conditions (minijets, Pythia events, color glass condensate) with a detailed comparison to data. Furthermore we will investigate the full impact parameter dependence of the transverse energy in order to understand elliptic and transverse flow at RHIC. Can the inelastic interactions generate almost the seen elliptic flow v_2, as implied by the exemplaric first calculations? How close are the calculations compared to (ideal) hydrodynamics? How close to reality? Also the partonic jet-quenching picture can be analysed in 3-D details. One can also compare the present calculations with some fixed and specified hydrodynamical initial conditions directly with calculations based on viscous relativistic hydrodynamics, either assuming Bjorken boost

invariance within an expanding tube or for full 3+1 dimensions. Such a comparison can tell how viscous the QGP really turns out to be.

The ultimate aim is to obtain a consistent picture of the (kinetic) QGP dynamics and to potentially 'deduce' the optimal initial condition of freed partons. If succesful, one can than extrapolate to future experiments at the much higher energies at the LHC.

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